

# **A FAILURE POLYMORPHISM THEORY FOR SYSTEM RELIABILITY MODELING CONSIDERING FAILURE MECHANISMS CORRELATIVITY**

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**Abstract:** There is a ‘polymorphism’ phenomenon in biology, which means individuals evolve to two or more clearly different phenotypes in the same population of a species, as a result of internal species gene and external environment condition. A similar ‘failure polymorphism’ phenomenon exists in complex systems. The failure modes in a complex system originate from different failure mechanisms. Failure mechanism is the root cause of product failure, and experts are used to paying attention to failure dependence problem by studying the failure behavior, few researches is conducted from the failure mechanisms correlation perspective. In this paper, polymorphism related concepts are incorporated into reliability field, new definitions of failure polymorphism are discussed, and Bayesian framework is introduced to tackle the problems of uncertainty factors and dynamic evaluation. Based on physics of failure (PoF), the paper presents a system-level reliability evaluation method considering the mechanisms correlativity.

**Key words:** failure polymorphism; failure mechanisms correlativity; physics of failure; Bayesian framework

**1 Introduction:** Failure mechanism is the root cause of product failure. It describes the internal physical and chemical process during products’ lifetime and reflects the relationship between internal variables and product life under certain circumstances. Plenty of engineering practices indicates that all failure scenarios of complicated system, which consists of numerous units (electronic component, material and structural part, etc.), could be finally traced back to several dependent failure mechanisms of certain units. One single unit may have different failure mechanisms in variant environment conditions. Moreover, different units may have the same failure mechanism in equivalent environment and units of the same sort may fail differently due to individual differences caused by processing technics [1]. One mechanism of certain unit may affect other mechanisms in the same unit, further one

unit fails may have an influence on other dependent units in the same system, all these cases will aggravate the degradation of system until it fails [2]. Experts are used to paying attention to fault dependence problem by studying the failure behavior (common caused failure and failure propagation), few researches is conducted from the failure mechanisms correlation perspective.

There is a ‘polymorphism’ phenomenon in biology, which means individuals evolve to two or more clearly different phenotypes in the same population of a species. This phenomenon occurs frequently in hierarchical and social insect species (such as termites, aphid, bee, etc.), as a result of internal species gene and external environment condition. Differentiated individuals focus on their specified work, which is called polytheism, to improve work efficiency and strength the dependences among individuals, help the species adapt to the environment changes ultimately. The essence of polymorphism phenomenon is a nature selection process from microcosmic to macrocosmic, in which the species produces adaptive differentiation on individuals to keep surviving in dynamic environment, and the differentiated representations of individuals promote the evolution of the whole species in turn.

A similar ‘failure polymorphism’ phenomenon exists in complex systems. Hierarchical system is composed of various units in different levels, and different unit failure modes originated from diverse failure mechanisms may lead to different failure mode of system [3]. Assumptions in biologic theory are introduced for failure polymorphism, the internal factors of which are the uncertainty of units’ properties (material, structural design, process level, etc.), while the external factors are the uncertainty of environment and load conditions. Failure mechanisms of unit that are susceptible to these factors may have intense reciprocal effects, and result in the unit failure polymorphism, which means the same unit may have different kinds of failure modes. The interactions among different units’ failure modes generated by their functional or structural dependences are supposed as the failure polytheism. As a consequence of the small changes in system configuration, the reliability of system begins to decline, which can be regarded as the evolution of system failure state. In this paper, polymorphism related concepts are incorporated into reliability field, new definitions of failure polymorphism are discussed, and Bayesian framework is introduced to tackle the problems of uncertainty factors and dynamic evaluation. Based on physics of failure (PoF), this paper presents a system level reliability evaluation method considering the mechanisms correlativity.

**2 Method for System Level Reliability Evaluation Considering the Mechanisms Correlativity:** In a complex system, parts, components and subsystems have its own property and behavior, which can affect the final state of the system [4]. In order to study the similar ‘failure polymorphism’ phenomenon in complex systems,

biological polymorphism phenomenon is introduced to reliability field. As shown in Figure 1, taking the micro electro mechanical system (MEMS) as an example, property and behavior of the parts, components and subsystems are redefined.

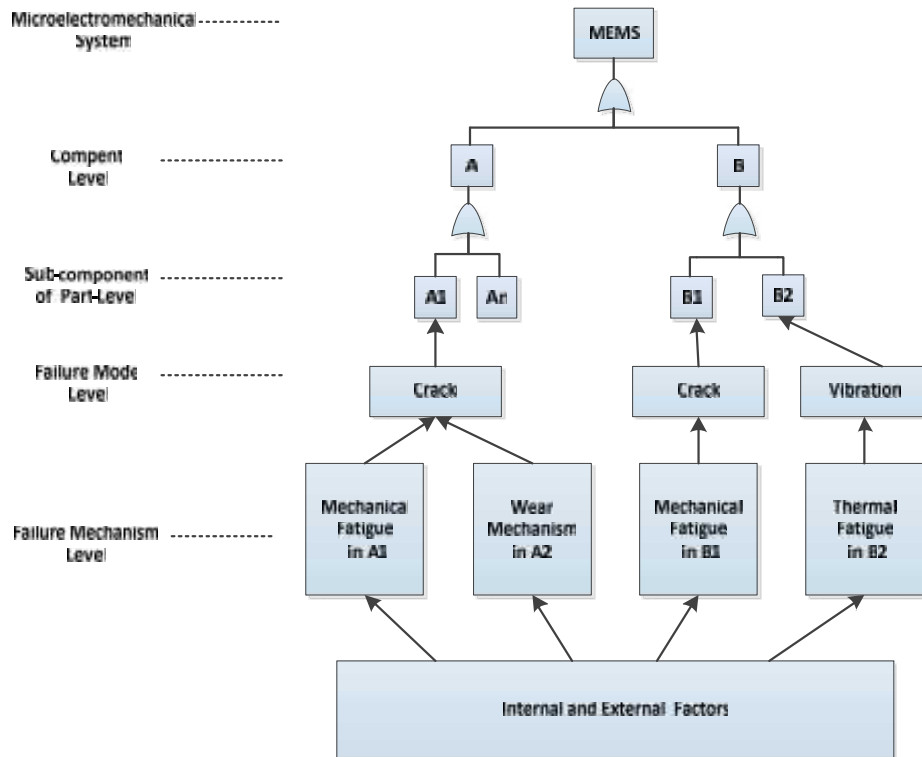


Figure 1: The Structure Diagram of MEMS

**2.1 New Definitions of Failure Polymorphism:** There is a ‘polymorphism’ phenomenon in biology, which means individuals evolve to two or more clearly different phenotypes in the same population of a species. Similarly, in a complex system, parts, components and subsystems will show a variety of failure modes, which is called ‘failure polymorphism’ phenomenon. For example, in the MEMS, both the sub-component A1 and B1 may cause crack failure, while the sub-component B2 may generate mechanical vibration [5].

The ‘polymorphism’ phenomenon in biology is mainly due to the differentiation of biotic population. And the causes of the differentiation include hereditary and non-hereditary factors. Analogously, the ‘failure polymorphism’ phenomenon is resulted from the differentiation of parts, components and subsystems. The reasons for differentiation including the differences among individuals and the effects of the environment, which is called failure differentiation factor.

There is biological adaptation in ‘polymorphism’ phenomenon. Biological adaptations means the organism can adjust themselves to the environment. In

‘failure polymorphism’ phenomenon, failure can also adapt to the environment. For example, in the MEMS (Fig. 1), the sub- component A1 will initiate crack (or crack propagation) to fit the combined effects of alternating mechanical stress and corrosion. The MEMS will eventually degeneration or failure to fit the harsh external environment.

In the MEMS, there is correlativity in the failure adaptation, such as the correlativity between the two failure mechanisms in sub-component A1. The failure of A1 is due to the combined effects of mechanical fatigue and corrosion. The failure between sub-components B1 and B2 are both affected by external alternating mechanical stresses. The correlativity is due to the same environment and working conditions. In addition, the same design, material, position or other factors will result failure correlativity. Correlativity also means the degradation of a part may be affected by the failure mechanism of other parts. It is great difficulties to evaluate the reliability of system level by considering the uncertainty factors and failure mechanism correlativity. In this paper, Bayesian framework is introduced to tackle the problems of uncertainty factors and dynamic evaluation [6]. Based on physics of failure (PoF), this paper presents agent oriented modeling method to evaluate the system reliability considering the mechanisms correlativity.

**2.2 Single Mechanism Modeling Under Bayesian Framework:** Based on PoF, a component has certainty parameters which mean the component has a determined life. However, for a number of components, their processing quality, structural size and material properties are uncertainty. Meanwhile, environmental stress and other parameters of components in service are also uncertainty, which will result the life of components becoming uncertain. Here, based on PoF, the uncertainty of these parameters is taken into account by the Bayesian updating, which makes the life evaluation result closer to the actual engineering situation.

Take the sub-component B1 as an example. As a mechanical unit of the MEMS, B1 has the main failure mode of crack propagation and the corresponding failure mechanism is mechanical fatigue failure. The physics of failure model in B1 is shown in Equation (1).

$$N_B = \frac{1}{K_B S_B^n} \quad (1)$$

Where  $N_{B1}$  is the number of cycles before failure;  $S_{B1}$  is the stress amplitude;  $K_{B1}$  is the proportional constant and  $n$  is the mechanical parameter.

By fitting the accelerated test data, the lifetime  $N$  of the unit B1, is given a logarithmic normal distribution:  $N_{B1} \sim LN(\mu_{B1}, \sigma_{B1})$ . So the mean value  $\mu_{B1}$  can be represented by Equation (2).

$$\mu_B = \log\left(\frac{1}{K_B S_B^n}\right) \quad (2)$$

Therefore, the life-stress multivariate function of unit B1 is shown in Equation (3).

$$f(N_{B1} | S_{B1}, K_{B1}, n, \sigma_{B1}) = \frac{1}{\sigma_{B1} N_{B1} \sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{\ln(N_{B1}) + \ln(K_{B1}) + n \ln(S_{B1})}{\sigma_{B1}} \right)^2} \quad (3)$$

The parameter vector of the model is shown in Equation (4).

$$\theta = (K_B, n, \sigma_B)^T \quad (4)$$

The uncertainty parameter  $\sigma_{B1}$  has no prior information, and assume the parameter  $\sigma_{B1}$  follows a uniform distribution from -3 to 3 in the Bayesian analysis. By investigating the literature, the prior information matrix of the parameters  $K_{B1}$ ,  $n$  are shown in Equation (5).

$$\theta_{K,n} = \begin{bmatrix} K & n \\ 2.06E-11 & 4.59 \\ 6.11E-14 & 4.42 \\ 3.00E-10 & 2 \\ 5.13E-10 & 3.61 \\ 1.13E-10 & 2 \\ 4.53E-10 & 2.09 \end{bmatrix} \quad (5)$$

By fitting the two parameters in the priori information matrix with a normal distribution, we can get the prior distribution of the parameters,  $K_{B1} \sim \text{Normal}(2.33E-10, 2.41E-10)$ ,  $n \sim \text{Normal}(3.10, 1.19)$ ,  $\sigma_{B1} \sim \text{Uniform}(-3, 3)$ .

The data of Bayesian updating is derived from the accelerated fatigue tests where the stress amplitude  $S_{B1}$  is a constant. The test data  $D_{B1}$  contains  $F$  complete failure observations  $t_i$  and  $H$  right delete measured value  $T_j$ , namely  $D_{B1}\{(t_i, T_j), i = 1, \dots, F, j = 1, \dots, H\}$ .

The Bayesian update process uses the test data  $D_{B1}\{(t_i, T_j), i = 1, \dots, F, j = 1, \dots, H\}$ . The likelihood function of the life-stress model is shown in Equation (6).

$$\begin{aligned} L(D_B | K_B, n, \sigma_B) &= L(t_1, \dots, t_F, T_1, \dots, T_H | K_B, n, \sigma_B) \\ &= \prod_{i=1}^F f(t_i | K_B, n, \sigma_B) \prod_{j=1}^H R(T_j | K_B, n, \sigma_B) \end{aligned} \quad (6)$$

Where  $R(T_j | K, n, \sigma)$  is the reliability equation, which is expressed by Equation (7).

$$R_B(T_j | K_B, n, \sigma_B) = 1 - \int_0^{T_j} f(x | K_B, n, \sigma_B) dx \quad (7)$$

According to the Bayesian equation, the posterior distribution of the model parameters is shown in Equation (8).

$$P(K_B, n_B | D_B) = \frac{L(D_B | K_B, n_B, \sigma_B) \pi_0(K_B, n_B, \sigma_B)}{\int \int \int L(D_B | K_B, n_B, \sigma_B) \pi_0(K_B, n_B, \sigma_B) dK_B dn_B d\sigma_B} \quad (8)$$

The probability distribution of the life-stress model of sub-component B1 is shown in Equation (9).

$$f_B(N_B, S_B) = \int \int \int f(N_B | S_B, K_B, n_B) P(K_B, n_B | D_B) dK_B dn_B d\sigma_B \quad (9)$$

The Markov chain Monte Carlo (MCMC) method is used to obtain the analytic solution of Equations (8) and 9 by WinBUGS [7]. After running the WinBUGS code, the posteriori distribution of the uncertainty parameters  $K_{B1}$ ,  $n$ , and  $\sigma_{B1}$  are shown in Figure 2.

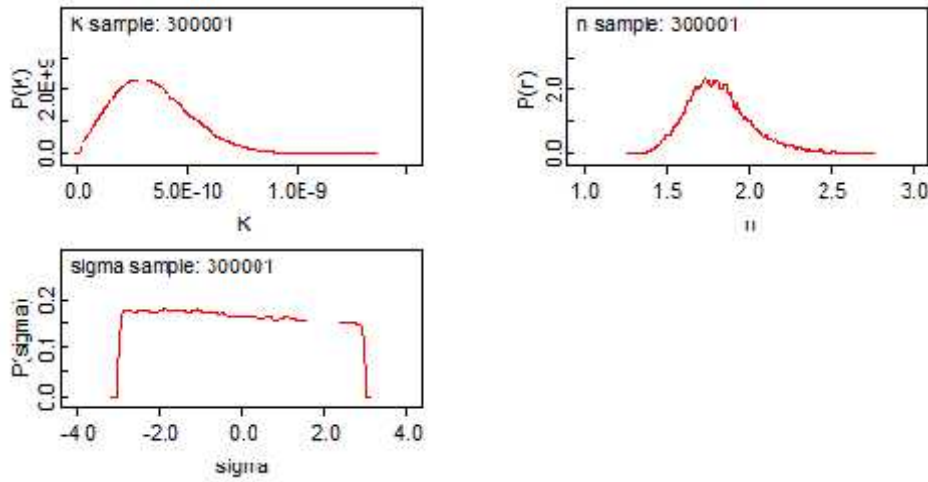


Figure 2: The Posteriori Distribution of Uncertainty parameters  $K_{B1}$ ,  $n$ , and  $\sigma_{B1}$

Table 1 shows the final results of the Bayesian updating process for the uncertain parameters  $K_{B1}$ ,  $n$ , and  $\sigma_{B1}$ .

Table 1: The Final Result of  $K_{B1}$ ,  $n$ , and  $\sigma_{B1}$  after 300000 Bayesian Parameters Update

Node	$\mu$	$\sigma$	MC error	2.5%	median	97.5%
$K_{B1}$	3.445E-10	1.986E-10	4.894E-12	6.758E-11	3.271E-10	7.299E-10
$n$	1.815	0.198	0.006776	1.49	1.794	2.278
$\sigma_{B1}$	0.017	1.613	0.02243	-2.858	0.023	2.88

According to the stress-life model, the 300000 groups of posterior data  $K_B$ ,  $n$ , and  $\sigma_B$  are simulated by Monte Carlo method. And the probability density distribution of the life distribution in sub-component B1 can be obtained. There is no need to know the life distribution of the sub-component B1. Each underlying failure data  $N_B$  will be used as input of the upper level. And then we can use the data to simulate the degradation process of the system. It can be seen that the probability physical model obtained under the Bayesian framework can take full account of the material and environmental uncertainty [8]. So it is more objective and closer to the actual

situation. Obviously, the modeling processes and steps of the single failure mechanism in sub-component B2 are similar to sub-component B1.

**2.3 Considering the Mechanisms Correlativity for System Reliability Modeling Based on Multi-Agents:** At present, only a few deterministic models involve the interaction of the failure mechanisms. And there is no probabilistic failure physical model considering the interaction between failure mechanisms. System-level modeling will be difficult because of the diversity of its components and the interaction between their failure mechanisms. Here we propose a hybrid modeling method based on multi-agents and system dynamics to solve the problems. In order to ensure that there is a failure mechanism correlativity in the studied system and meanwhile the system is simplified as much as possible, only the sub-components A1 and B1 in the MEMS are considered.

In the MEMS, each component, part, and subsystem is treated as an agent [9]. In response to internal and external changes, each agent is created with a single output variable whose value is a function of one or more input variables. Each input variable represents another agent, pointing to the output variable. The generic formula for the output variables is shown in Equation (10).

$$Y = f(X_1, X_2, \dots, X_n) \quad (10)$$

Where Y represents the function of the output variable in the agent, consisting of the input variables  $X_1, X_2, \dots, X_n$ . The functional equation  $f(X)$  is a probabilistic failure physical model of the output in the agent, which has been determined in Section 2.2. Such agents are also called "multi-agents" because their evolution relies on input variables from external stress and the interaction among internal agents. There may be such agents that cannot get any input from other agents, which refers to the 'self-contained agents'. The development of the self-contained agents is independent of other agents, so they have independent output variables in the form of probability density functions.

As the uncertainty of  $f(X)$  and  $X_i$ , the output in each agent is always stochastic. The output in each self-contained agent is also stochastic and consistent with the appropriate probability density function (modified or indeterminate parameter). Any feature or parameter given by a constant value should be treated as a constant instead of an agent [10]. A determined value, which can change in a known trend as the system evolves (such as time variables) or according to the time equation defined, need to be assigned as an agent.

For example, a case study of agent B1, the output of the agent B1 (the number of cycles) is related to the parameters  $K_{B1}$ , n, and  $\sigma_{B1}$ . The posterior probability distribution of  $K_{B1}$ , n, and  $\sigma_{B1}$  is acquired by the Bayesian update in Section 2.2. So

the parameters  $K_{B1}$ ,  $n$ , and  $\sigma_{B1}$  belong to agents. The parameter  $S_{B1}$  is not an agent as the parameter is a constant. According to Equation (1), the relationship between the output and each input in B1 can be determined. As shown in Figure 2, the result of the output is uncertainty with the form of probability density.

The output of the single failure mechanism in the sub-component A1 is similar to the sub-component B1. Besides, the interaction between the fatigue failure mechanism and the wear failure mechanism in A1 can be expressed by the interaction among the multi-agents. Meanwhile, the output of the agent A1 is the length of the crack. In this case, it is necessary to add a monitoring agent  $M_{A1}$  to monitor the crack length in real time. Once the crack length reaches the failure threshold, the monitoring agent  $M_{A1}$  is activated and output the total failure time  $N_A$ . Then the value of  $M_{A1}$  becomes 0 and continue to monitor the next time.

The failure of sub-components A1 and B1 can cause degradation in the MEMS. Therefore, sub-components A1 and B1 need monitoring agents  $MD_A$  and  $MD_B$  to represent the system degradation caused by the failure of A1 or B1. Besides, a monitoring agent  $M_S$  is added to monitor the current degradation of the MEMS. When the system reaches the degradation threshold, the monitoring agent  $M_S$  will be activated and output the failure time  $N_S$  of the MEMS. The specific relationships are shown in Figure 3.

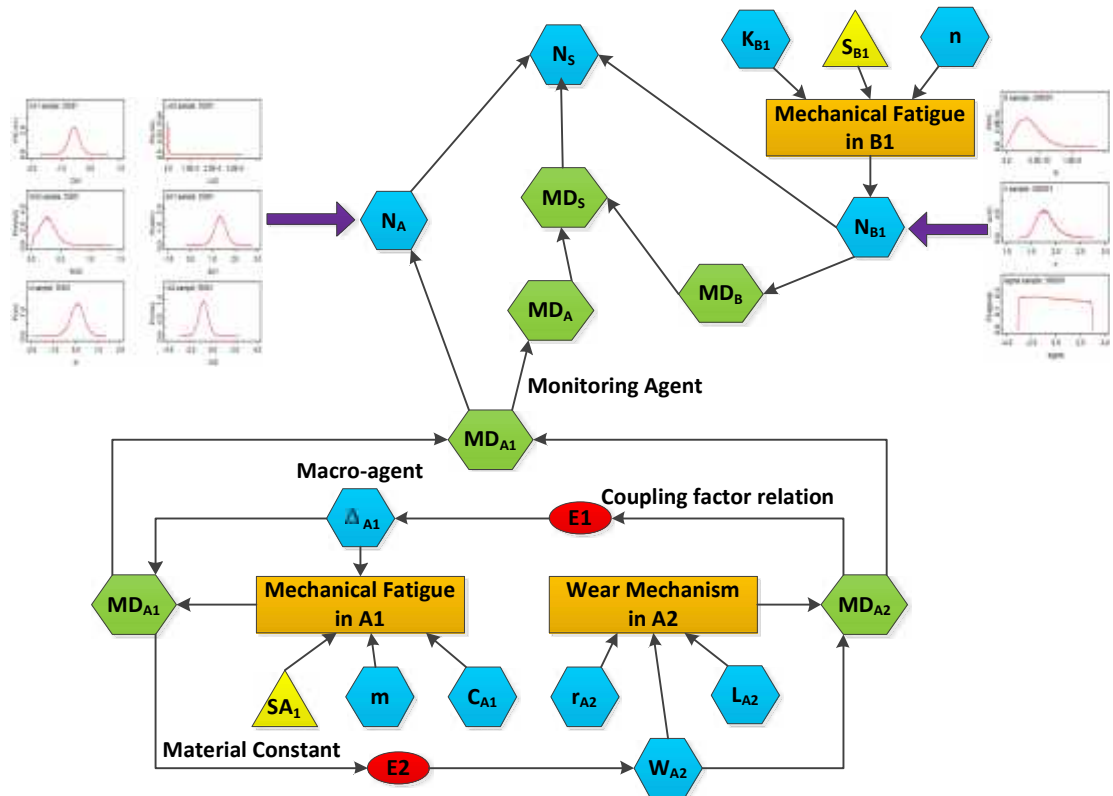


Figure 3: MEMS Model based on Multi-Agents



In Figure 3, there is a coupling relationship between the fatigue failure mechanism and the wear failure mechanism in sub-component A1. On the one hand, the damage caused by wear will affect the stress intensity factor  $K_{A1}$  in the fatigue failure mechanism and the agent E1 represents the coupling relation. On the other hand, the generation of fatigue cracks will affect the wear rate  $W_{A2}$  in the wear failure mechanism and the agent E2 represents the coupling relation. Thus the modeling of the MEMS can deal with the correlativity among different failure mechanisms. According to the system dynamics and characteristics in the MEMS, the system degradation amount  $M_A$  caused by the failure of the sub-component A1 and the system degradation amount  $M_B$  caused by the failure of the sub-component B1 can be expressed by the Equation (11).

$$MD_S = g(MD_A, MD_B) \quad (11)$$

When the system degradation amount  $MD_S$  reaches the failure threshold, the total failure time  $N_S$  of the MEMS is output. Depending on the total failure time, the remaining life of the MEMS can be evaluated. This modeling method considers not only the uncertainty of the parameters such as the process and structure in the underlying materials, but also the correlativity between different failure mechanisms, which will be very promising.

**3 Case Study:** The paper presents probability physics of failure model in single failure mechanism about sub-component B1. And meanwhile the paper presents a system-level reliability evaluation method considering the mechanisms correlativity based on multi-agents. The following are the simulation results.

**3.1 Life Evaluation of the Single Failure Mechanism Under Bayesian Framework:** The probabilistic failure physical model construction method of a single failure mechanism is analyzed in Section 2.2. After updating the Bayesian parameters in Section 2.2, Monte Carlo simulation is used to evaluate the life of sub-component B1 through the posteriori data parameters  $K_{E1}$ ,  $n$  and  $\sigma_{E1}$ . According to Equation (1) and the posterior parameters, the probability density distribution histogram of the lifetime  $N_B$  can be obtained, and then connecting the midpoint in each rectangle block, we can get the probability density curve of the lifetime distribution about sub-component B1. Monte Carlo simulation results are shown in Figure 4. It can be seen that the distribution of the lifetime is closest to the posterior distribution of the uncertainty parameter  $K_{E1}$ , so the dispersion parameter  $K_{E1}$  is the main parameter that causes the lifetime to be dispersed.

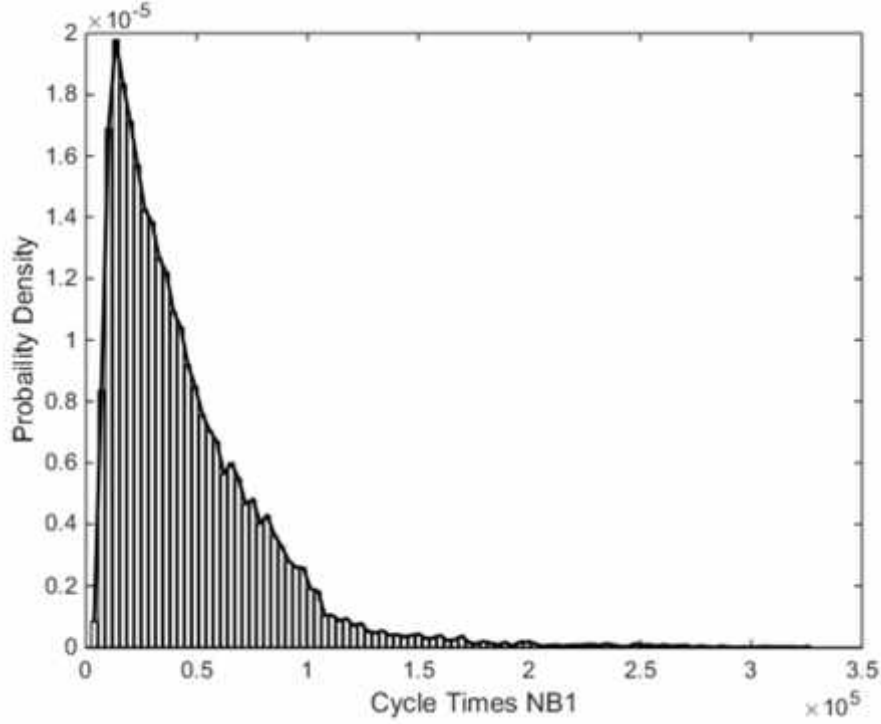


Figure 4: The Lifetime Probability Density Histogram and Probability Density of Sub-component B1

This method of modeling under the Bayesian framework combines the priori information with the test data. Meanwhile, the method considers the dispersion of factors, such as the material properties, dimensions and stress, adding these dispersions to the existing failure physical model. Under Bayesian framework, the probability of failing physical model will be achieved, and thus the result will be more accurate and closer to the actual situation.

**3.2 Remaining Life Evaluation of MEMS Based on Multi-Agents:** In Section 2.3, the MEMS modeling method based on Multi-agents is described in detail. When we measure the degradation of the sub-components and the number of load cycles, the remaining life of the MEMS can be evaluated based on multi-agents under Bayesian framework [11]. In the test, we get three sets of data, including the degradation of the sub-components and the number of load cycles  $N_i$ . The data  $D_{\alpha}$  can be expressed in Equation (12).

$$D_{\alpha} = \{MDA_i, MDB_i, N_i, i = 1, 2, 3\} \quad (12)$$

The remaining life of the MEMS is represented by NL, and then NL can be represented by Equation (13).

$$NL_i = N_S - N_i \quad (13)$$

According to the three sets of data, the multi-agents model will be updated under Bayesian framework, and the obtained data will be fitted to evaluate the remaining life of the MEMS. The results are shown in Figure 5.

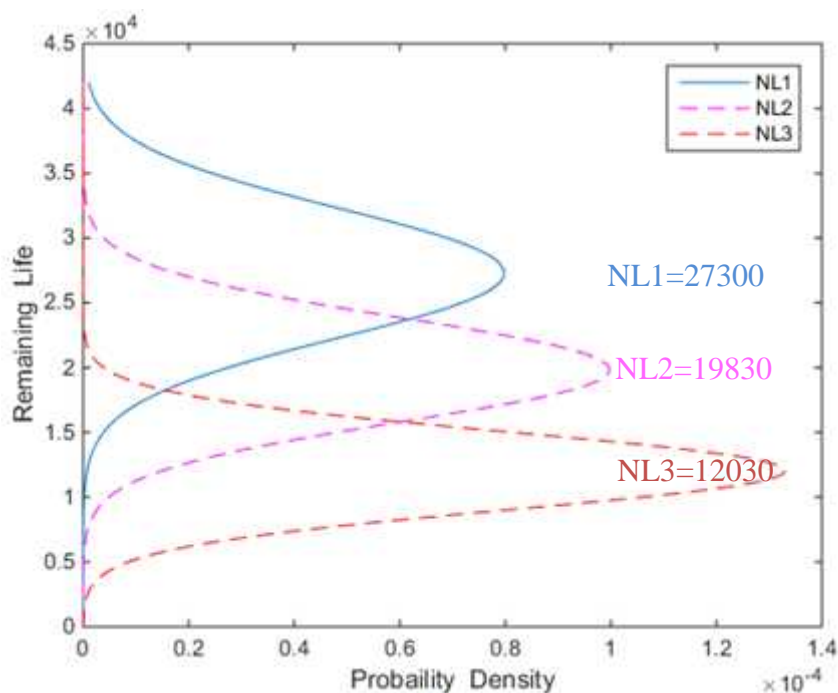


Figure 5: The Remaining Life of MEMS

The data obtained from Bayesian updating are fitted by MATLAB, and we can find that the lifetime distribution of MEMS is normal distribution. The type of life distribution in B1 is different from MEMS, which can be shown that the life distribution type of the sub-component is not directly related to the life distribution type of the whole system.

In Figure 5, NL1, NL2 and NL3 respectively represent the remaining life of the MEMS system under the first, second and third measurements. Since the lifetime distribution of the MEMS system conforms to the normal distribution, the expected value of the remaining life can be expressed by the mean of the normal distribution. The mean of NL1, NL2 and NL3 are 27300, 19830 and 12030 respectively. It can be seen that the expected value of the remaining life in MEMS decreases with the increase of the load cycle number.

Since the lifetime distribution of the MEMS system conforms to the normal distribution, the variance represents the life uncertainty. The three sets of data are used to update the Bayesian parameters, and then NL1, NL2 and NL3 are becoming narrower, which means that their variance is getting smaller. That indicates the uncertainty of remaining life NL1, NL2 and NL3 in the MEMS are getting smaller. After each Bayesian update, the multi-agents modeling will integrate the test

information, and thus the uncertainty of system remaining life assessment is declining. The more data are measured, the more information will be integrated in multi-agents modeling and the remaining life assessment will be more accurate.

**4 Conclusions:** In this paper, polymorphism related concepts are incorporated into reliability field, new definitions of failure polymorphism are discussed, and Bayesian framework is introduced to tackle the problems of uncertainty factors and dynamic evaluation. Under the Bayesian framework, based on multi-agents, the reliability evaluation method of system-level is realized considering the correlativity between the failure mechanisms. Compared with other traditional methods, the method in the present paper will be very promising and has the following advantages:

(1) The posterior parameters under the Bayesian framework can fully consider the influence of the lower units including material, technology and environmental load uncertainties. Based on probability physics of failure, the life assessment method is more accurate and close to the actual situation.

(2) With the help of biological polymorphism, the reliability evaluation method of system-level can be realized considering the failure mechanisms correlativity. At the same time, this evaluation method can transfer the uncertainty of the bottom units to the system-level through the real-time information exchange among the agents, and finally realize the dynamic simulation of the system.

(3) The measured data can be used to update the Bayesian parameters, so that the uncertainty of the system will be reduced in the simulation process, which is conducive to the more accurate assessment to the remaining life of the system.

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