

A Novel Approach for Machinery Health Prognostics Using Statistical Tools

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Abstract: Condition based maintenance of machinery is being much talked about in the engineering sector of defense and commercial industry. A lot of expenditure is generally incurred on condition monitoring of machinery to avoid unexpected downtimes and failures vis-à-vis optimizing machinery operation. The concept is ever evolving due to technological advancements as well as with the emergence of unique nature of defects. Vibration Analysis is a potent tool of condition monitoring for prediction and diagnostics of machinery failures. Presently, time and frequency spectra are being widely used for defect diagnostics of machinery. However, they require signal conditioning to eliminate noise and to enhance resolution of spectrum. Extensive research in the area of signal processing has been undertaken to refine time and frequency spectra. Notwithstanding application of statistical tools for analysis of various defects in machinery using condition monitoring data can be a viable option. Research in this area, where statistical models have been applied, revealed encouraging results. In this paper, we have modeled bearing vibration data by applying time varying Markov Switching Auto Regressive method which was found very helpful in estimating RUL of machinery.

Key words: Autoregressive; Defect frequency; Forecasting; Markov switching; Prognostics; Regime, Time variant

Introduction: In real life, machinery operates in diversified conditions. These conditions include environment, adopted maintenance philosophy, operators/maintainer skills, operating conditions etc. These operating conditions govern the machinery operation and health; machines have similar released life by the manufacturer tend to fail at different operational hours.

Prognostic on the basis of physical models is theoretically fine but its practical manifestation in real time environment is difficult, as machinery life is governed by various known and

unknown effects. The effects generated on machinery during operations, at times, deprive the implementation of physical laws due to multifarious physical changes being encountered.

On the other hand, prognosis using data driven models, carries better practical manifestation in real life. Condition monitoring data provides a comprehensive understanding of machinery condition under prevailing operating conditions. This data while providing various condition indications of machinery can be manipulated using different tools for prognosis. Taking lead from huge usage in the fields of finance and medical, data driven techniques have become popular approaches for prognosis in engineering sector also. A comprehensive review [1] shows that many of the prognostics designs are based on data driven models.

Data driven methods are broadly divided in to two approaches i.e. Artificial Intelligence (AI) and Statistical approaches. AI approach has its unique computational power; however, it requires an extensive expertise. Comparatively, statistical methods are relatively easier to adopt and apply. Furthermore, it can be deduced from comparative analysis [2] that statistical tools offer an easier and reliable prognostics framework. Similar reviews for different scenarios already presented also support the above argument [3], [4].

Statistical data driven methods include many linear and nonlinear methodologies. These broadly include regression models, wiener processes, gamma processes, markovian models, stochastic filtering based models, covariate based hazard models etc. Comprehensive discussion on various statistical models with their suitability of application is discussed in [5].

The nature of machinery operation involving nonlinearity and randomness is challenging and carries modeling complexities for the estimation of machinery life. Nonlinearity in case of machinery can be a defect, whose propagation will be highly random under prevailing operating and environmental conditions. In term of statistics, the nature of this damage propagation will be stochastic. The situation becomes more complex when there are several nonlinearities (defects), evolving randomly with time under varying operating conditions [4].

Researchers have made enormous endeavors to estimate the machinery remaining useful life focusing on nonlinearities and randomness discretely or in combination. Particle filter method has been adopted by various researchers. This method provides more accuracy as compared to other algorithms. It has a good capability to model nonlinear dynamical systems with multivariate data [6],[7],[8] . Hidden Markov Models [9],[10] is a good tool for performing fault and degradation diagnosis on random dynamic systems with multi failure modes specially when failure state is unobserved. It can model different stages of failures and can distinguish fault types in a component.

Wiener process also referred to as Brownian motion apply concept of random walk of particle relating with machinery performance to measure the degradation. However, when we talk about the RUL estimation especially in case of a nonlinear phenomena then the Wiener method has to assume the mean degradation path as linear; hence the dynamics of nonlinearity cannot be adjudged efficiently [5], [11], [12]. Regression based

models are common tools used for time series analysis as well as for RUL estimation due to their simple application [5]. These models utilize various condition monitoring independent variables regressed against some dependent variable to estimate the trend and subsequent calculation of RUL. The application of this tool is enough matured in the field of economics, finance etc [13]. Other techniques including Principle Component Analysis (PCA), Gaussian Mixture Model, Logistic Regression, Statistical pattern recognition, Kalman filter, Match matrix, Support vector machine have been applied and discussed for the machinery prognostics with detail in [1], [2], [3], [4].

In this paper we will discuss the application of Time Variant Markov Switching Autoregressive Model to estimate remaining useful life considering Roller element bearing (REB) as a case.

Bearing Prognosis: Roller Element Bearings (REB) are the most critical component of machinery and a determinant of machinery health. The loading of machinery either internal or external is supported by the bearings. Most of the condition monitoring tools primarily focused on bearing health; especially vibration technology which evolved on bearing health dynamics.

REBs are meant to eliminate sliding contacts between surfaces by use of rolling elements. REBs may have balls or rollers and accordingly they are classified. The REBs are normally classified on the basis of No of balls/rollers, ball/rollers diameter, inner/outer race diameter, pitch diameter etc. The four parts of REBs are balls/rollers, inner race, outer race and cage as illustrated in ‘Figure 1’.

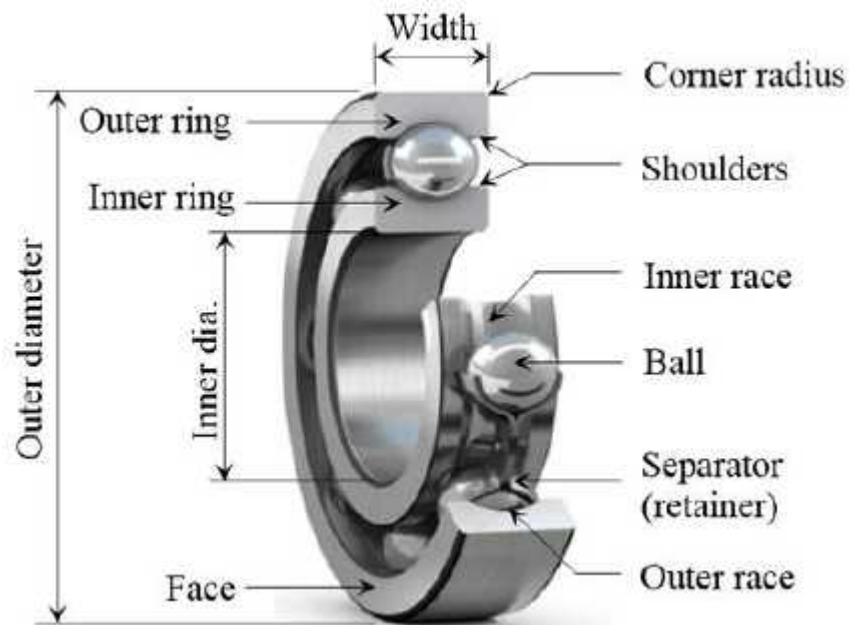


Figure 1: Deep Grooved Ball Bearing

Wear in bearings arises due to stresses in rolling contact surfaces. Under normal operating conditions with no defect, the wear process is linear. However, when there is

some abnormality induced like abnormal operating conditions, defects (misalignment, foreign particle ingress, lubrication problems etc), then the stress level rises to enhanced metal to metal contact causing various parts of bearings to deteriorate. This deterioration results in exponential trend of bearing wear. The defects of bearings are seen as defect frequencies in the vibration spectrum as Fundamental train frequency or cage frequency (FTF), Ball pass frequency outer race (BPFO), Ball pass frequency inner race (BPFI) and Ball spin frequency (BSF).

Dynamic models for bearing wear evolution have been studied by various researchers. These include the modeling to study the effects of Elasto-Hydrodynamic lubrication, excessive clearances, brinelling phenomena, non-uniform loading conditions, modulation effects etc. Moreover, frequency response in multi degree of freedom systems using FEM and other mathematical tools has been under focused in various studies [14]. Beside physical models, data driven models are also adopted by various researchers [15]. The pros and cons of physics based and data driven based models have already been discussed in preceding lines under introduction; same are also relevant for bearing application.

Experimental setup: Test rig is a simple arrangement driven by a 2 Hp variable speed induction motor. A customized designed shaft is then supported by two bearings. The test bearing UC-203 (deep groove ball bearing) follows the support bearings which will be subjected to various radial and axial loading conditions, under modified housing. A screw type loading mechanism is installed for static load in radial direction and for axial loading a load is mounted on shaft see 'Figure 2'

Testing is done on the basis of varying loading and variable speed conditions. The concept of accelerated testing is adopted in order to perform the failure analysis with varying static load from 0N-75N and a constant axial load of 100N. The analysis is confined to the bearing only, in view of its criticality. The data collection was conducted using NI DAQ system.



Figure 2: Experimental Setup

Model and Method: Markov switching model explained by [16] is studied to account for nonlinearities in financial time series data and the auto regressive component to cater for the non stationary process in the data. The model provides a appropriate methodology to describe changes in the dynamic behaviour of machinery health data. It assumes the presence of a finite number of states, whereby the parameters are allowed to take on different values with regard to the regime prevailing at certain point of time. The regime shifts arise from the outcome of an unobserved random variable which is assumed to evolve following Markov chain. Consideration of time varying regime probabilities also make this model practically suitable for scenarios where condition are not constant and varying with time.

Model specification: Let y_t represents machinery health variable that undergo changes over time. Considering this series as a subset of stochastic process whose probability distribution depends on the realization of a hidden discrete stochastic process z_t . The stochastic process y_t is directly observable, whereas z_t is a latent random variable that is visible only indirectly through its effect on the identification of y_t . The changes in health variable across different regimes are modeled by the regime switching “Equation (1)”:

$$z_t = \beta_{s_t} + \sum_j^k \beta_{j,s_t} z_{t-j} + \epsilon_t \quad (1)$$

here ϵ_t is normally distributed term, so that $\epsilon_t \sim N(0, \sigma_{s_t}^2)$ and ‘ s_t ’ unobserved discrete state variable. Whereas parameter (β_{j,s_t}) depends on state variable $S_t = i, i = 1, 2, \dots, N$ which represents regime in process at time t .

Statistical inference and estimation procedure: The MARKOV Switching Regression model extends the simple exogenous probability framework by specifying the first order Markov process for the regime probabilities followed by likelihood computation, filtering, and smoothing.

Regime probabilities: The first order Markov assumption necessitates that the likelihood of being in a regime depends on the preceding state as in “Equation (2)”, so that:

$$P(s_t = j | s_{t-1} = i) = p_{ij}(t) \quad (2)$$

Normally, these probabilities are assumed to be time-invariant in order that $p_{ij}(t) = p_{ij} \forall t$. These probabilities in transition matrix form are shown in “Equation (3)”:

$$p(t) = \begin{bmatrix} p_{11}(t) & \cdots & p_{1M}(t) \\ \vdots & \ddots & \vdots \\ p_{M1}(t) & \cdots & p_{MM}(t) \end{bmatrix} \quad (3)$$

here ij -th component signify the probability of transitioning from regime ‘ i ’ in period $t-1$ to regime ‘ j ’ in period t . Since each row of the transition matrix indicates a full set of conditional probabilities so a different multinomial specification for each row ‘ i ’ is required which is defined in “Equation (4)”:

$$p_i(z_{t-1}, \delta_i) = \frac{e^{-z_{t-1}^i \delta_i}}{\sum_{i=1}^M e^{-z_{t-1}^i \delta_i}} \quad (4)$$

$$f_j = 1, \dots, M \quad a_i = 1, \dots, M \quad w_h = n \quad \delta_{t_i} = 0$$

In Hamilton model [16], the regime probabilities are time invariant; however, in real life the probabilities are not constant in time rather they are time variant. In case of defect evolution in machinery it may happen in case of non stationary conditions that the defect growth may enhance due various loading/operating conditions in certain point of time. In contrast to that let $[\mathbf{S}_t]_{t=1}^T$ illustrate the first order sample path then its two state Markov process with time varying transition probabilities are explained by [17],[18] and represented in ‘‘Equation (5)’’:

$$\begin{array}{c} \text{Time } t \\ \text{State 0} \qquad \qquad \qquad \text{State 1} \\ \left[\begin{array}{cc} P_t^0 & P_t^0 = (1 - P_t^1) \\ P(S_t=0|S_{t-1}=0, z_{t-1}, \beta_0) & P(S_t=1|S_{t-1}=0, z_{t-1}, \beta_0) \\ \frac{e^{-z_{t-1}^0 \beta_0}}{1 + e^{-z_{t-1}^0 \beta_0}} & \frac{1 - \frac{e^{-z_{t-1}^0 \beta_0}}{1 + e^{-z_{t-1}^0 \beta_0}}}{1 + e^{-z_{t-1}^0 \beta_0}} \\ \hline P_t^1 & P_t^1 \\ P(S_t=0|S_{t-1}=1, z_{t-1}, \beta_1) & P(S_t=1|S_{t-1}=1, z_{t-1}, \beta_1) \\ \frac{1 - \frac{e^{-z_{t-1}^1 \beta_1}}{1 + e^{-z_{t-1}^1 \beta_1}}}{1 + e^{-z_{t-1}^1 \beta_1}} & \frac{e^{-z_{t-1}^1 \beta_1}}{1 + e^{-z_{t-1}^1 \beta_1}} \end{array} \right] t-1 \quad (5) \end{array}$$

Here

$$z_{t-1} = (1, z_{1,t-1}, \dots, z_{(k-1),t-1})^t$$

$$\beta_t = (\beta_{t0}, \beta_{t1}, \dots, \beta_{t(k-1)})^t \text{ and } i = 0, 1$$

Likelihood Evaluation and Filtering: The likelihood contribution for given observation is computed by utilizing density function in each regime by the one step ahead probability of being in that regime as shown in ‘‘Equation (6)’’:

$$L_t(\beta, \gamma, \sigma, \delta) = \sum_{m=1}^M \frac{1}{\sigma_m} \phi\left(\frac{z_t - \mu_t(m)}{\sigma(m)}\right) \cdot P(S_t = m | \mathfrak{I}_{t-1}, \delta) \quad (6)$$

$\beta = (\beta_1, \dots, \beta_M)$, $\sigma = (\sigma_1, \dots, \sigma_M)$, δ are parameters that determines the regime probabilities $\phi(\cdot)$ is the standard normal density function and \mathfrak{I}_{t-1} is the information set in period lag one. Besides in undemanding form δ represents the regime probabilities themselves.

The full log-likelihood is normal combination of

$$l(\beta, \gamma, \sigma, \delta) = \sum_{t=1}^T l_t \left\{ \sum_{m=1}^M \frac{1}{\sigma_m} \phi\left(\frac{z_t - \mu_t(m)}{\sigma(m)}\right) \cdot P(S_t = m | \mathfrak{I}_{t-1}, \delta) \right\} \quad (7)$$

‘‘Equation (7)’’ can be maximized w.r.t $(\beta, \gamma, \sigma, \delta)$

Filtering process estimates the updated probabilities which are computed by utilizing Bayes' theorem and the laws of conditional probability which can be expressed as "Equation (8)":

$$P(s_t = m | \mathfrak{S}_t) = P(s_t = m | z_t, \mathfrak{S}_{t-1}) = \frac{f(z_t | s_t = m, \mathfrak{S}_{t-1}) P(s_t = m | z_t, \mathfrak{S}_{t-1})}{f(z_t | \mathfrak{S}_{t-1})} \quad (8)$$

The right side of the expression (8) is obtained as a by-product of densities obtained during likelihood estimation. On substitution we have "Equation (9)":

$$P(s_t = m | \mathfrak{S}_t) = \frac{\frac{1}{\sigma_m} \left(\frac{z_t - \mu_r(m)}{\sigma(m)} \right) \cdot p_m(G_{t-1}, \delta)}{\sum_{j=1}^M \frac{1}{\sigma_j} \left(\frac{z_t - \mu_r(j)}{\sigma(m)} \right) p_j(G_{t-1}, \delta)} \quad (9)$$

where G_{t-1} represents vector of exogenous variables.

Smoothing: Estimates of the regime probabilities may be improved by using all of the information given in the sample. The smoothed estimates for the regime probabilities in period t use the information set in the final period, T , in contrast to the filtered estimates which employ only contemporary information, \mathfrak{S}_t . Intuitively, using information about future realizations of the dependent variable $y_s (s > t)$ improves estimates of being in regime m in period t , because the Markov transition probabilities link together the likelihood of the observed data in different periods. Another approach for smoothing requires only a single backward recursion through the data. Under the Markov assumption, the joint probability is given by "Equation (10)" and "Equation (11)":

$$P(s_t = i, s_{t+1} = j | \mathfrak{S}_T) = P(s_t = i | s_{t+1} = j, \mathfrak{S}_T) \cdot P(s_{t+1} = j | \mathfrak{S}_T) \quad (10)$$

$$= \frac{P(s_t = i, s_{t+1} = j | \mathfrak{S}_T)}{P(s_{t+1} = j | \mathfrak{S}_T)} P(s_{t+1} = j | \mathfrak{S}_T) \quad (11)$$

"Equation (12)" shows the smoothed probability in period t obtained by marginalizing the joint probability with respect to \mathfrak{S}_{t-1} :

$$P(s_t = i | \mathfrak{S}_T) = \sum_{j=1}^M P(s_t = i, s_{t+1} = j | \mathfrak{S}_T) \quad (12)$$

Initial Probabilities: The Markov Switching Filter requires initialization of the filtered regime probabilities in period zero, $P(s_0 = m | \mathfrak{S}_0)$. There are few ways to proceed with initial probabilities. Most commonly, the initial regime probabilities are set to the steady state values implied by the Markov transition matrix. The values are thus treated as functions of the parameters that determine the transition matrix. Alternately, prior knowledge may be used to specify regime probability values, and lastly for treating of the initial probabilities as parameters which are to be estimated.

Application of Model: The dynamic behavior of Machinery health variables (Z_t) observed over time is modeled by utilizing MASR with time varying transition probabilities. In this regards two state Markov switching model is estimated in which Z_t is regressed over switching coefficients Z_{t-1} and constant keeping AR(3) as non-varying regressor. Maximum likelihood estimated equations for different regime with standard deviation in parenthesis are shown “Equation (13)”:

$$Z_t = \begin{cases} 0.578 + 1.315 Z_{t-3} + [AR(3) = 0.343] + \epsilon_{t_1} & \text{Healty State} \\ (0.261) \quad (0.359) \quad (0.057) \\ 0.0821 + 0.813 Z_{t-3} + [AR(3) = 0.343] + \epsilon_{t_2} & \text{Unhealty State} \\ (0.013) \quad (0.027) \quad (0.057) \end{cases} \quad (13)$$

with $\epsilon_{t_1} \sim N(0, 0.085)$ and $\epsilon_{t_2} \sim N(0, 0.057)$

‘Equation (13)’ demonstrates that differences in the health state specific mean indicate the growth of defects in machinery for the period under study. The probabilities values for both regimes are significant ($p < 0.05$) which confirm that dynamics in both regimes are substantial.

The time varying transition probability matrix of the predicted ‘Equation (13)’ is presented in ‘Table 1’ which indicates that the probability of switching from healthy state to unhealthy state is 0.4066, while remaining in healthy state is 0.5933.

Table 1: Time Varying Transition Probability Matrix.

	Healthy State	Unhealthy State
Healthy state	0.593398	0.406602
Unhealthy State	0.048505	0.951495

The variations in the estimates of time varying transition probabilities for the constructed model in different states is presented in ‘Figure-3’ which reflect that machinery remain in its healthy for more hours.

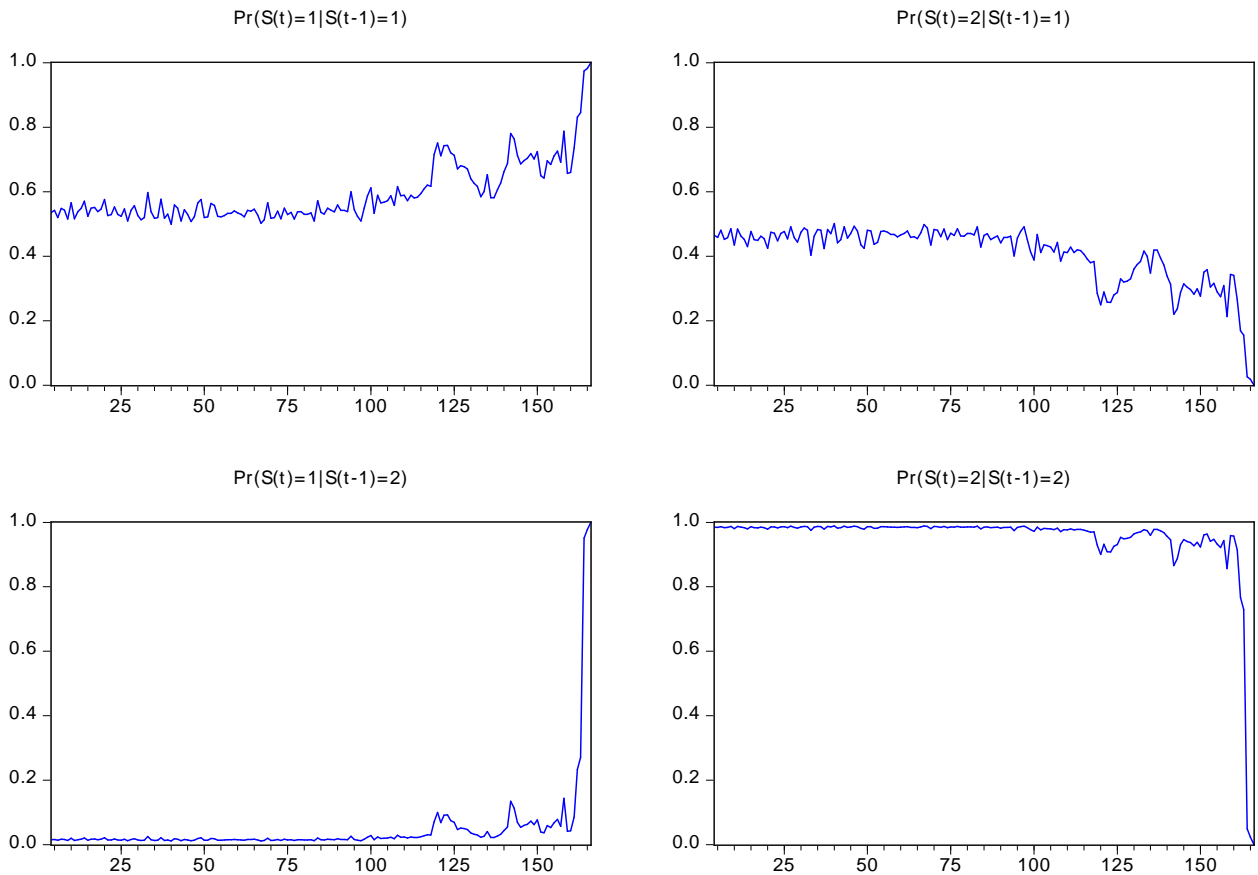


Figure 3. Fluctuation in Time-varying Markov Transition Probabilities

The subsequent time varying expected durations depicted under ‘Table 2’ imply that machinery remain in the healthy state for a longer duration before moving to unhealthy state.

Table 2: Time Varying Expected Duration Matrix.

	<u>Healthy State</u>	<u>Unhealthy State</u>
Mean	76.08617	49.90062
Std. Dev.	931.9972	23.58307

Based on above expected duration, the expected variation point between healthy & unhealthy states is shown in ‘Figure 4’.

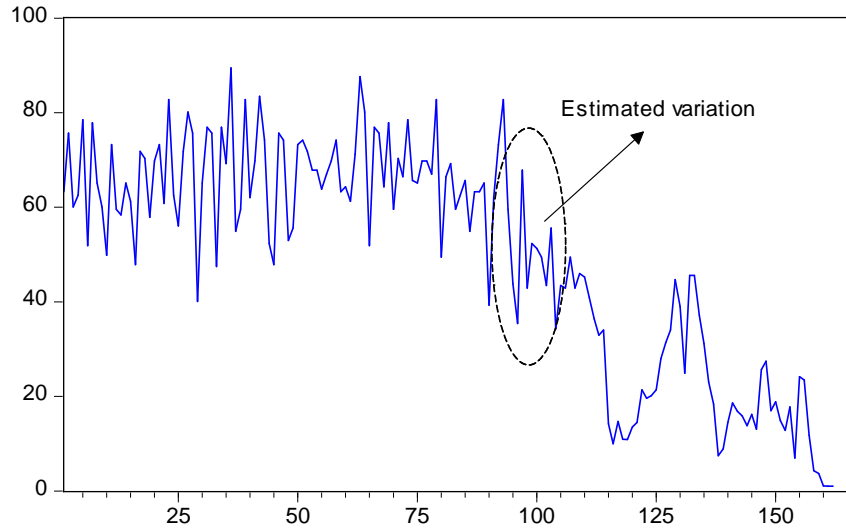


Figure 4. Expected variation between two states

'Figure 5' exhibit the filtered high & low variance probabilities of being in the two health states. The filtering process keeps on updating the estimated probability in order to determine the likelihood of machinery health from one regime to the other. It is evident from graph that machinery unhealthy states begin after 120 running hours.

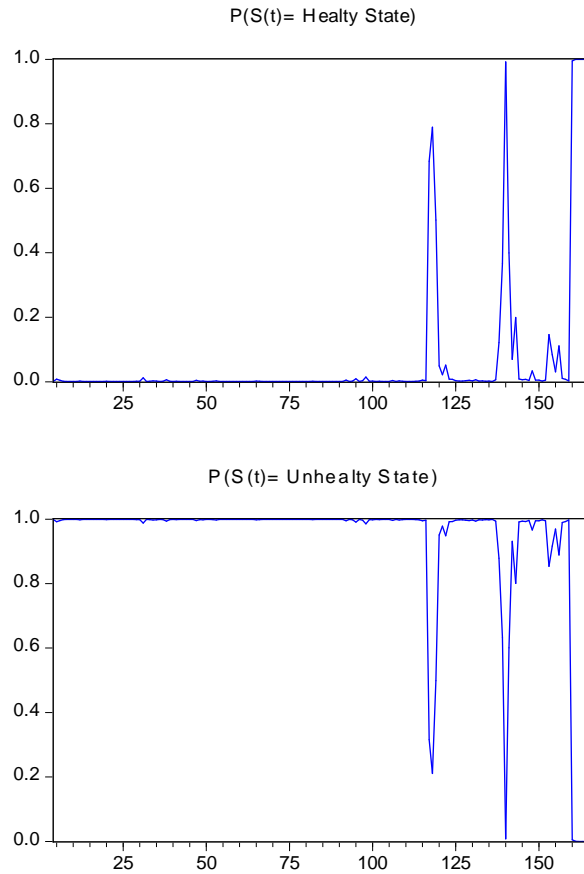


Figure 5. Filtered high & low variance probability plot.

'Figure 6' shows the real time instability in actual and forecasted health indicator which demonstrates inconsistency in machinery health after its usage of about 120 running hours and it keeps on growing afterwards. Its continuous handling without maintenance will distort the mechanism after 164 running hours.

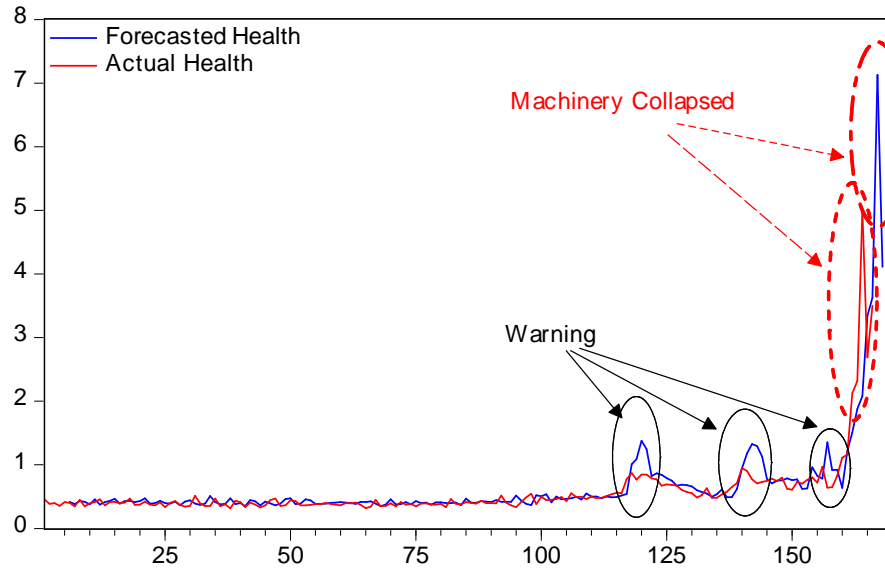


Figure 6. Actual and forecasted health indicator signifying volatility in health parameter

The forecasted performance indicator with its estimated statistics are illustrated in 'Table 3' which authenticates the predicted model:

Table3. Forecasting performance statistics

Forecasting Performance Indicator	Estimated Statistics
RMSE	0.279202
MAE	0.108040
Theil Inequality Coefficient	0.183851
Bias Proportion	0.004359
Variance Proportion	0.098221

Conclusion:

In this communication two state of time-varying Markov-switching model is utilized to machinery health variable as it permits the shift likelihood to fluctuate thoroughly with information predictors which reflects the future health state of machinery. This investigation signifies that machinery stayed in healthy state for a period of 116 hours before becoming unhealthy. The real time performance of the machinery is also forecasted by employing the estimated model. The forecasted performance statistics signify the capability of the constructed model. Finding of this study will serve as a basis for making suitable decision for machinery future operation.

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Biographies:



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2. Senior Assistant Professor Dr. Asif Mansoor has PhD in Applied Mathematics from University of Karachi and is presently in Post graduate teaching faculty at Bahria University Karachi Campus. He has 12 international and 06 national research publications to his credential besides he has supervised 08 of students for research. He is regularly invited by eminent national universities and others to guide doctoral students on research methodologies, simulation and modeling and current trends in mathematics. His main research interest includes Probabilistic Method in Data Analysis, Stochastic Processes and CFD.



3. Qaisar Ali received Masters Degree in Thermo Fluids from NUST in 2010. He has over 07 years of research experience and 15 years Reliability Maintenance Management experience of wide range of industrial equipment/ machinery. He is certified Project Management Professional from Project Management Institute (PMI), USA. His 05 research papers have been published/ presented in International Journal/ Conferences. His areas of research interest are Heat transfer and fluid mechanics analysis across Porous Materials, Renewable Energy.